

NO CALCULATORS ALLOWED
SHOW PROPER CALCULUS-LEVEL WORK & SIMPLIFY ALL ANSWERS
(ANSWERS WITHOUT SOLUTIONS WILL NOT EARN FULL CREDIT)

For the following questions, you may use the formulae for $\frac{d}{dx} \sinh x$ and/or $\frac{d}{dx} \cosh x$ without proving them. SCORE: 6.5 / 6 PTS

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

a) Without using the logarithmic formula for $\cosh^{-1} x$, prove the formula for $\frac{d}{dx} \cosh^{-1} x$.

Let $y = \cosh^{-1} x$. Since $\cosh^2 y - \sinh^2 y = 1$, $x = \cosh y$
 $X = \cosh y$ ($y > 0$). $\sinh y = \sqrt{\cosh^2 y - 1}$, since $y > 0$, then $\sinh y > 0$.
 $\Rightarrow \sinh y = \sqrt{x^2 - 1}$ **BONUS 1**

$\frac{dy}{dx} \cdot \cosh y = \frac{d}{dx}(x)$
 $\frac{dy}{dx} \sinh y = 1$ ①
 $\frac{dy}{dx} = \frac{1}{\sinh y}$ ① $\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 1}}$ ②

b) Prove the formula for $\frac{d}{dx} \coth x$.

$\coth x = (\tanh x)^{-1}$
 $= \frac{\cosh x}{\sinh x}$

$\frac{d}{dx} \coth x = \frac{d}{dx} \left(\frac{\cosh x}{\sinh x} \right)$

$\Rightarrow \left(\frac{\cosh x}{\sinh x} \right)'$
 $= \frac{\sinh x \cdot \sinh x - \cosh x \cdot \cosh x}{\sinh^2 x}$
 $= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$ ①
 $= \frac{-1}{\sinh^2 x}$ ② $= -\operatorname{csch}^2 x$ ③

Find $\lim_{x \rightarrow -\infty} \coth x$. Do NOT use a graph. Give BRIEF algebraic or numerical reasoning.

SCORE: 3 / 3 PTS

$\lim_{x \rightarrow -\infty} \coth x$

$= \lim_{x \rightarrow -\infty} \frac{e^{2x} + 1}{e^{2x} - 1}$ ①

$= -1$ ②

If $\tanh x = -\frac{1}{3}$, find $\sinh x$.

SCORE: 2 / 4 PTS

You may use any hyperbolic identities from your textbook without proving them.

We have $1 - \tanh^2 x = \operatorname{sech}^2 x$.

Since $\tanh x < 0$, and $\cosh x$ always > 0 .

$$1 - \left(-\frac{1}{3}\right)^2 = \operatorname{sech}^2 x$$

$$\Rightarrow \sinh x < 0.$$

$$\operatorname{sech}^2 x = \frac{8}{9} \quad (1)$$

$$\Rightarrow \cosh^2 x = \frac{9}{8}$$

$$\Rightarrow \sinh x = -\frac{\sqrt{2}}{4} \quad (1)$$

We have $\cosh^2 x - \sinh^2 x = 1$.

$$\Rightarrow \frac{9}{8} - \sinh^2 x = 1$$

$$\Rightarrow \sinh^2 x = \frac{1}{8}$$

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook.

SCORE: 4 / 4 PTS

NOTE: This is NOT a question about derivatives.

Let $y = \sinh^{-1} x$.

~~We have $\cosh^2 y - \sinh^2 y = 1$~~

$$x = \sinh y.$$

$$\Rightarrow \cosh y = \sqrt{1 + \sinh^2 y}$$

$$x = \sinh y$$

$$\Rightarrow x = \frac{e^y - e^{-y}}{2} \quad (1)$$

$$e^y - e^{-y} = 2x.$$

$$(e^y)^2 - 2x(e^y) - 1 = 0 \quad (1)$$

$$e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2} \quad (1)$$

$$= x \pm \sqrt{x^2 + 1} \quad (1) \quad \text{Since } y > 0 \Rightarrow e^y = x + \sqrt{x^2 + 1} \quad (1)$$

$$y = \ln(x + \sqrt{x^2 + 1}) \quad (1)$$

Find $\frac{d}{dx} x^3 \sinh^{-1}(x^2)$. Simplify your final answer.

SCORE: 3 / 3 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$(\sinh^{-1} x)' = \frac{1}{\sqrt{x^2 + 1}} \cdot (x^2 + 1)^{-\frac{1}{2}}$$

$$\frac{d}{dx} x^3 \sinh^{-1}(x^2) = 3x^2 \sinh^{-1}(x^2) + x^3 \left[\frac{1}{\sqrt{x^2 + 1}} \cdot 2x \right]$$

$$= 3x^2 \sinh^{-1}(x^2) + 2x^4 \cdot \frac{1}{\sqrt{x^2 + 1}}$$

$$\frac{d}{dx} x^3 \sinh^{-1}(x^2) = 3x^2 \sinh^{-1}(x^2) + \frac{2x^4}{\sqrt{x^2 + 1}} \quad (1)$$

$$= 3x^2 \sinh^{-1}(x^2) + \frac{2x^4}{\sqrt{x^2 + 1}} \quad (1)$$